HEAT-EXCHANGE BOILING CRISIS IN VERTICAL CHANNELS OF DIVERSE GEOMETRY, PLUGGED AT THE BOTTOM

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A theoretical relationship is proposed for the determination of critical heat flows in long vertical tubes, and in channels of annular and rectangular configurations.

The boiling crisis which occurs in a channel closed off at the bottom (dead end) is caused by the loss of stability in the counterflow motion of the steam in the channel and by the liquid entering the channel from a collector above. The boundaries of stable counterflow motion of a gas and a liquid have been studied most fully for the case of adiabatic flows and most of the studies neglect the influence of mass exchange between the phases [1-5] in determining the conditions for the onset in the heat-exchange crisis. Some authors associate this heat-exchange crisis with the transition from the plug flow regime to a frothing regime [1], while others link this crisis with the transition from the frothing regime to an annular regime [2], or from an annular regime to a dispersive-annular regime [3].

The transition from the plug regime to the frothing regime, and from the annular regime to the dispersive-annular regime, is governed by the appearance of unstable waves at the boundary of phase separation. With counterflow motion the existence of such waves leads either to the capture of a portion of the liquid by a stream of ascending gas or to the separation of the entire runoff film. For long vertical tubes this phenomenon, known as "flooding," in the case of 6 < Bo < 15 and Re_1 > 300 is sufficiently well described by the relationship of Wallace [6]:

$$W_1^* (D)^{1/2} + W_2^* (D)^{1/2} = C_1, \tag{1}$$

where C_1 is an empirical coefficient that is dependent on the structural execution of the inlet and outlet from the working section. For tubes with pointed rear sections C_1 = 0.725, while for those whose sterns are rounded $C_1 \approx 0.88$, and if the end effects can be neglected, then C_1 = 1.

In the case of Bond numbers greater than 15, the critical reduced velocities become smaller than those determined from Eq. (1), while with Bo > 50, the "flooding" in long tubes is described by the following relationship [4]:

$$K_1^{1/2} + K_2^{1/2} = (3.2)^{1/2}. \tag{2}$$

Wallace [6] achieved the only analytical derivation of Eq. (1) for vertical tubes on the basis of a model of separated cylindrical phase flows. The application of this model to an annular channel leads to Eq. (1) in which the magnitude of the double clearance [7] is used in the place of the tube diameter. However, according to experimental data [1, 8], the reduced critical phase velocities in the annular channel are independent of the equivalent diameter, and it is noted in [8] that the characteristic linear dimensions of the annular channel in the description of the "flooding" is the channel perimeter. Nevertheless, the relationship proposed in the above-cited paper does not reflect this fact. On the basis of experimental data, it is demonstrated in [9] that for a rectangular channel it is the width of the channel (the larger side) that is the characteristic linear dimension, and that the reduced velocities in the case of "flooding" do not depend on the clearance. Then, assuming that the wetted perimeter is the characteristic linear dimension for rectangular and annular channels, we obtain a generalization of the Wallace relationship:

$$W_1^* (\Pi)^{1/2} + W_2^* (\Pi)^{1/2} = C_1/\pi^{1/4}, \tag{3}$$

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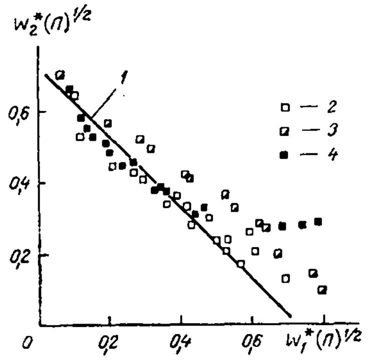


Fig. 1

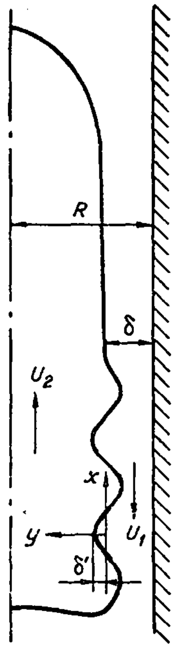


Fig. 2

Fig. 1. "Flooding" in rectangular channels: 1) calculation according to (3) when $C_1 = 0.725$; experiment [9]; 2) channel 40×1.5, 3) 40×2.4, 4) 40×5 mm².

Fig. 2. Waves at the side surface of a plug.

in which π has been introduced for the transition to Eq. (1) in the case of a circular tube. As we can see from Fig. 1, this relationship offers a good description of the experimental data in [9] with respect to the "flooding" in rectangular channels.

When a saturated liquid boils in a dead-end channel the reduced velocity of the steam exhibits its greatest value at the outlet from the heated segment and is determined from the heat-balance equation:

$$W_2 = N/rF\rho_2. \tag{4}$$

In this case, in the steady precrisis state a balance is achieved between the mass flow rates of the phases, and when the removal of the liquid by the core of the steam is neglected this balance is described by the equation

$$\rho_1 W_1 = \rho_2 W_2. \tag{5}$$

It follows from (3)-(5) that the most favorable conditions for "flooding" are realized at the outlet from the channel and the boiling heat-exchange crisis for a saturated liquid is described by the following criterial expression:

$$K_2(1 + (\rho_2/\rho_1)^{1/4})^2 = C_1^2 \sqrt{\mathrm{Bo}(\Pi/\pi)}, \tag{6}$$

where according to the definition of the criterion of hydrodynamic two-phase flow stability and according to Eq. (4):

$$K_2 = \frac{N}{rF(\rho_2^2 g\sigma(\rho_1 - \rho_2))^{1/4}}.$$

It follows from Eq. (6) that K_2 is proportional to $\Pi^{1/2}$ and is independent of the clearance of the rectangular and annular channels, which is in agreement with the data of [5]. Nevertheless, the results of [10] indicate the existence of a pronounced relationship between the criterion K_2 and the clearance of the annular channel for large ratios of the diameter to the clearance.

To describe the transition from the influence on the magnitude K_2 of the perimeter of the annular channel to the influence of its clearance, we will assume that the heat-exchange crisis is brought about by the losses in stability for the plug flow regime, which corresponds to visual observations for boiling in thermosiphons that are nearly full [11]. Destruction of the plug is associated with the separation of the liquid phase in its trailing portion and the formation of an extremely perturbed wake [6]. The appearance of unstable one-dimensional capillary waves at the vertical side surface of the plug (Fig. 2) is determined by the dispersion relationship for the inviscid motion of the liquid film [12]:

$$\rho_1(\Omega - nU_1)^2 \operatorname{cth}(n\delta) + \rho_2(\Omega + nU_2)^2 \operatorname{cth}(n(R-\delta)) = \sigma n^3, \tag{7}$$

where Ω and n are the frequency and wave number.

The equation for the phase-separation surface:

$$\delta' = \delta_1 \exp\{i(nx + \Omega t)\}. \tag{8}$$

In the general case, the determined frequency is a complex quantity

$$\Omega = \omega + \beta i. \tag{9}$$

By means of the equality

$$(U_1 - U_2)^2 = \frac{\sigma n}{\rho_2} \operatorname{th}(n(R-\delta)) \left[1 + \frac{\rho_2 \operatorname{th}(n\delta)}{\rho_1 \operatorname{th}(n(R-\delta))}\right], \tag{10}$$

which, according to (8) and (9), determines the conditions for the appearance of waves of increasing amplitude, we can change over from the real roots to the complex. Generally, $\rho_2 \ll \rho_1$ and $\delta < R/2$, and we can neglect the second terms in the brackets.

To determine the velocities of phase motion resulting in the appearance of waves that grow with time, in addition to Eq. (10) we also need a relationship for the length of the wave achieved in the channel. The length of the wave was determined in [13] on the basis of the principle of minimum dissipation of the relative film energy, and the application of this principle for channels of complex geometry raises the need for a numerical solution of the three-dimensional problem. In this paper we therefore propose to determine the length of the wave on the basis of the familiar experimental fact that in the motion of a solitary plug in a nonviscous liquid through a tube that is closed off from above, waves will be observed in the trailing end of the plug, and the velocity of these waves $C = \omega/n$ is equal to the limit velocity of plug flotation [14]:

$$C = U_\infty. \tag{11}$$

Then, in accordance with (7), the wave number will be determined by the equation

$$\sigma n = \rho_1 (U_\infty + U_1)^2 \operatorname{cth}(n\delta_0), \tag{12}$$

where δ_0 is the thickness of the film on flotation of a single plug in a tube closed off from above.

Since the upper end is closed, we have

$$(1 - \varphi_0) U_1 = U_\infty \varphi_0, \tag{13}$$

where $\varphi_0 = (1 - \delta_0/R)^2$ is the true vapor content in the rear portion of the plug, which on the basis of an approximate analytical solution for the potential flotation of a single plug in a circular tube is equal to 0.684 [15].

With Bo(D) > 6 and $Re_1 > 300$ the velocity of plug flotation U_∞ in a circular tube is determined by the equation [6]:

$$U_\infty = 0.345 \sqrt{g(\rho_1 - \rho_2) D/\rho_1}. \tag{14}$$

In this case, according to Eqs. (12) and (13), cth $(n\delta_0) \approx 1$ and the wave number is equal to

$$\sigma n = \rho_1 U_\infty^2 / (1 - \varphi_0)^2, \tag{15}$$

i.e., the length of the waves in the trailing portion of the plug is diminished as the tube diameter increases:

$$\lambda = 2\pi \left(\frac{1-\varphi_0}{0.345}\right)^2 \frac{l^{*2}}{D}. \tag{16}$$

The length of the waves depends only on the relative velocity of gas and liquid motion (the flotation rate) and is independent of the velocity of the mixture and, according to (10) and (15), the condition for the breaking up of the rear portion of the gas plug is described by the equation

$$(U_1 - U_2)^2 = \frac{\rho_1 U_\infty^2}{\rho_2 (1-\varphi_0)^2} \operatorname{th}\left[\left(\frac{0.345}{1-\varphi_0}\right)^2 \operatorname{Bo}(D)\operatorname{Bo}(R-\delta)\right]. \tag{17}$$

To calculate the critical heat flows for the case of boiling in channels closed from below, the average velocity of phase motion must be expressed in terms of the flow-rate quantities, i.e., the reduced velocities.

In the case of turbulent plug-type flow [6]:

$$U_2 = C_0 (W_1 + W_2) + U_\infty, \tag{18}$$

where C_0 is an empirical coefficient that is a function of channel geometry and in the case of a circular tube is equal to 1.2.

According to definition, the reduced velocity of the liquid in the film is

$$W_1 = U_1 (1 - (1 - \delta/R)^2) \approx 2U_1 \delta/R. \tag{19}$$

With turbulent film flow [6]

$$\delta/R = 0.126 (W_1^*)^{2/3}. \tag{20}$$

After a number of simple transformations, through utilization of (18)-(20), Eq. (17) is brought to the following form:

$$C_0\left(\frac{W_2}{U_\infty}\right) + C_0\left(\frac{W_1}{U_\infty}\right) + 2.0\left(\frac{W_1}{U_\infty}\right)^{1/3} + 1 = \frac{1}{(1-\varphi_0)}\sqrt{\frac{\rho_1}{\rho_2}} \tag{21}$$

with consideration of the fact that when $\operatorname{Bo}(D) \geq 6$ the expression for the hyperbolic tangent in Eq. (17) assumes a value equal to unity.

System of equations (5) and (21) uniquely defines the critical value of the reduced phase velocities in a dead-end channel. Variation of ρ_1/ρ_2 allows us to present the solution of this system of equations in the form of a curve on a plane $\left(\sqrt{\frac{\rho_2}{\rho_1}}\frac{W_2}{U_\infty};\ \frac{W_1}{U_\infty}\right)$, each point of which will correspond to the critical state for a certain relationship of densities.

As we can see from Fig. 3, the solution of Eqs. (5) and (21) with a change in ρ_1/ρ_2 from 16 to 1600 can be approximately described by the relationship

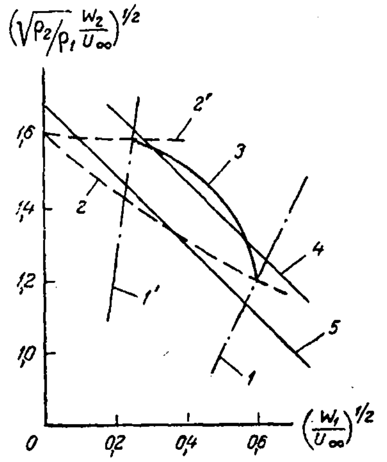
$$\left(\frac{W_1}{U_\infty}\right)^{1/2} + \left(\sqrt{\frac{\rho_2}{\rho_1}}\frac{W_2}{U_\infty}\right)^{1/2} = 1.15\,((1-\varphi_0)\,C_0)^{-1/2}. \tag{22}$$

With an increase in the tube diameter, the closure of the film about the channel perimeter ceases to affect the length of the wave which, as it diminishes [Eq. (16)], tends toward some finite value determined from the condition that as $\operatorname{Bo} \to \infty$ the "overturning" of the film flow ($U_1 = 0$) is described the criterial equality [3]

$$K_2 = 3.2. \tag{23}$$

From Eq. (10), with hyperbolic cotangents equal to unity and $\rho_2 \ll \rho_1$, we obtain the following conditions for the "overturning" of the film:

$$K_2 = (2\pi l^*/\lambda)^{1/2}, \tag{24}$$



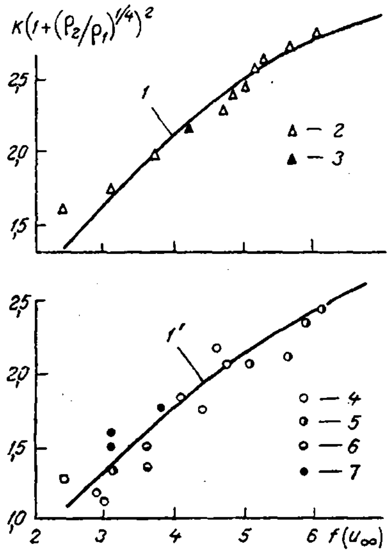


Fig. 3 Fig. 4

Fig. 3. Boundary of stability for plug flow regime in dead-end channels: 1, 1') Eq. (5) for ρ_1/ρ_2 = 16 and 1600; 2, 2') Eq. (21) for ρ_1/ρ_2 = 16 and 1600; 3) boundary of stability, solution of system (5) and (21) for ρ_1/ρ_2 = 16-1600; 4) relationship (22); 5) relationship (1) for C_1 = 1.

Fig. 4. Comparison of theoretical relationship with experimental data on the heat-exchange crisis in tubes and annular channels: 1, 1') calculation in accordance with (27) for C_1 = 0.75 and 0.68; 2) water, circular tubes [3]; 3) water, annular channel, D_1 = 23 mm and D_2 = 12 mm [3]; 4-7) tubes and annular channels; water, ethanol, carbon tetrachloride, hexane, respectively [5].

and, according to (23) and (24), as Bo → ∞:

$$\lambda_\infty = 0.6l^*.$$

We can assume that in the general case

$$\lambda^2 = \lambda_\infty^2 + (\lambda')^2, \tag{25}$$

where λ' is the wavelength for flotation of a small-diameter plug [Eq. (16)]. The losses in plug flow regime stability in the case of heat-carrier boiling in a tube closed from below when Bo(D) > 6 will then be described by the equation

$$\left(\frac{W_1}{U_\infty}\right)^{1/2} + \left(\sqrt{\frac{\rho_2}{\rho_1}}\,\frac{W_2}{U_\infty}\right)^{1/2} = 1.15C_1\left((1-\varphi_0)\,C_0\right)^{-1/2} \times \\ \times \left[1 + \left(\frac{C_1^2(1.15)^2}{3.2C_0(1-\varphi_0)}\,\frac{U_\infty}{\sqrt{g\Delta\rho l^*/\rho_1}}\right)^4\right]^{-1/8}, \tag{26}$$

which for a tube with D ≈ $7l^*$ coincides with the Wallace "flooding" relationship (1), and as Bo → ∞ changes into Eq. (2).

From system (4), (5), and (26) we obtain the criterial relationship defining the critical heat flows in the boiling of a saturated liquid in long dead-end channels:

$$K_2(1 + (\rho_2/\rho_1)^{1/4})^2 = C_1^2 f(U_\infty)\left(1 + \left(\frac{C_1^2 f(U_\infty)}{3.2}\right)^4\right)^{-1/4}, \tag{27}$$

where

$$f(U_\infty)=\frac{(1,15)^2}{(1-\varphi_0)\,C_0}\,\frac{U_\infty}{\sqrt{g\Delta\rho l^*/\rho_1}}$$

and for a tube $f(U_\infty) = 1.20\sqrt{Bo(D)}$.

Equation (27) is suitable for channels of various geometries, since it contains no numerical quantities that are characteristic exclusively of a circular tube. Then the influence of channel geometry on the critical heat flows is described by the relationship between K_2 and $f(U_\infty)$. In this case, if for C_0 and U_∞ there exist in the literature theoretical relationships [16, 17], then φ_0 has been determined only for a circular tube and, consequently, in first approximation it should be assumed that $\varphi_0 = 0.684$ for channels of any geometry.

The limit (steady-state) velocity of nonviscous flotation of a single plug in the case of $Bo(\Pi/\pi) > 6$ is determined from the equations [17]:

$$\mathrm{Fr}(\Pi/\pi)=\begin{cases} C_2 & \text{when } \Pi/a<77,\\ 8.8C_2\,(a/\Pi)^{1/2} & \text{when } \Pi/a\geqslant 77,\end{cases} \tag{28}$$

where $C_2 = 0.345 - 0.057D_2/D_1$, $C_2 = 0.288 - 0.031a/b$ for annular and rectangular channels, respectively.

Then $f(U_\infty)$ depends on the configuration of the channel in the following way:

$$f(U_\infty)=\begin{cases} 4.19\,\dfrac{C_2}{C_0}\sqrt{\mathrm{Bo}(\Pi/\pi)} & \text{when } \Pi/a<77,\\[2ex] 36.8\,\dfrac{C_2}{C_0}\sqrt{\mathrm{Bo}(a/\pi)} & \text{when } \Pi/a\geqslant 77.\end{cases} \tag{29}$$

It follows from formulas (27) and (29) that when $\Pi/a < 77$ the criterion K_2 is approximately proportional to $\Pi^{1/2}$, while when $\Pi/a \geq 77$ the criterion is proportional to $a^{1/2}$, which corresponds to experimental data from various authors [1, 5, 10].

As we can see from Fig. 4, relationship (27) describes the heat-exchange crisis in tubes and annular channels with an error that does not exceed 5% for the data contained in [3] [$3 \leq Bo(\Pi/\pi) \leq 30$, pressure 0.2-12 MPa] and 10% for the data of [5] [$Bo(a) \geq 0.54$; $4 \leq Bo(\Pi/\pi) \leq 35$; $3 \leq \Pi/a \leq 105$, pressure 0.1 MPa]. In this case, the slight difference in the magnitudes of C_1 is a result of the different structural execution of the outlet from the working section.

NOTATION

ρ_j, the density of the j-th phase; σ, the coefficient of surface tension; ν_1, the kinematic viscosity of the liquid; r, the latent heat of vapor formation; N, the thermal load; W_j, the reduced velocity of the j-th phase; D, the tube diameter; D_1 and D_2, the outside and inside diameters of the annular channels; a, the clearance of the annular or rectangular channel; b, the width of the rectangular channel; Π, the wetted channel perimeter; F, the area of the transverse cross section of the channel; ℓ, the characteristic linear dimension of the channel; λ, the wavelength; $n = 2\pi/\lambda$, the wave number; $\ell^* = \sqrt{\sigma/g(\rho_1 - \rho_2)}$, the capillary constant; $Bo(\ell) = \ell/\ell^*$, the Bond number; $K_j = W_j\sqrt{\rho_j}/\sqrt{g(\rho_1 - \rho_2)\ell^*}$, the Kutateladze criterion; $W_j^*(\ell) = K_j/Bo^{1/2}(\ell)$, the dimensionless reduced velocity; $Fr(\ell) = U_\infty\sqrt{\rho_1}/\sqrt{g(\rho_1 - \rho_2)\ell}$, Froude number; $Re_1 = \sqrt{g(\rho_1 - \rho_2)D/\rho_1}D/\nu_1$, the Reynolds number.

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THE EFFECT OF THE NONSTEADY STATE AND TURBULENCE ON INTERPHASE HEAT AND MASS TRANSFER IN THE RELATIVE MOTION OF BUBBLES IN A BOILING STREAM

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An analytical solution is proposed for the heat flow between vapor bubbles and a liquid with consideration of the nonsteady relative velocity of bubble motion in the nonsteady pressure field of a boiling stream.

Numerous papers have been devoted to the questions of rates of change in bubble dimensions in steady-state nonmoving volumes of a boiling liquid; reviews of these papers are presented, for example, in [1, 2]. However, to create closed mathematical models of nonequilibrium boiling flows we must have relationships which describe the transfer of heat and mass between moving vapor bubbles and a liquid, with consideration given to the definitive features of bubble evolution within the stream. At the present time, only individual special cases have been investigated.

A solution was obtained in [3] for the specific heat flow q between the vapor bubbles and a liquid, with consideration given to the relative nonsteady velocity of phase motion, while the quasisteady self-similar numerical approximation of that solution is presented in [4]. In [5] and in the works of Nakoryakov et al. [6] analytical solutions were obtained for q with consideration of the nonsteady nature of the pressure field in the process of bubble growth in the absence of any effect exerted by the induced convection that is due to the relative motion of the bubbles.

Semiempirical relationships have been derived in [7-9] and in these allowance is made for the decisive effect of turbulence on the specific heat flow between vapor bubbles and the liquid in a stream. These relationships in this case make no allowance for the relative motion of the bubbles, nor of the nonsteady nature of the flow parameters, and in the limit (an insignificant degree of flow turbulence) these relationships do not correspond to other special solutions.

In the general case we have the combined effect of all of the above-enumerated factors on the exchange of heat and mass between bubbles and liquid in a boiling stream. However, at the present time there exists no solution which allows for the nonsteady nature of the relative velocity of bubble motion in a field of nonsteady pressure for a boiling turbulent stream.

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